



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

quired length is 105.826 yards. Note here that if the weight is taken as distributed along the horizontal, i. e., along the x -axis instead of along the wire there is obtained by simple moments $c \, dy = x \, dx$ and with the origin as before except that the low point is taken instead of a point c below, there is obtained the parabolic relation $x = \sqrt{2cy}$, with $y = 1$. Hence, $x = \sqrt{2800} = 52.915$ and $L = 105.83$ yds., which is sufficiently close for this problem. As a matter of interest, it is seen that since $y/c = \cosh x/c$, the origin a distance c below the low point, by expanding the cosh to two terms, $1 + x^2/2c^2$, and replacing y by $y - c$ the parabolic relation above is obtained, which is a measure of the approximation of this method of solving the catenary.

(b) Here $y/c = \sec \varphi + \frac{1}{2}m \tan^2 \varphi$; $x/c = gd^{-1} \varphi + m \tan \varphi$. $m = 1H$, where 1 is the unit extension of the wire. Since the wire weighs 0.1 pound per yd., taking steel as weighing 10.2 pounds per sq. in. per yd., and the modulus of elasticity as 30,000,000 pounds per sq. in., we have since $1 = H/AE$, where a is the area of the section of the wire and E the modulus, $m = 6664 \times 10^{-5}$. Expressing the y equation as a quadratic in $\sec \varphi$ and solving for φ , $\varphi = 2^\circ 5' 54''$. Substituting this value in the equation for x and again replacing the gudermanian by its logarithmic equivalent we get $x = 54.74$ and $L = 109.58$ yds. Do not mistake the difference between this value and the above as the extension of the wire. Its meaning is that when allowance is made for extension it is necessary to spread the supports above the difference apart in order to maintain the deflection of one yard. The parabolic expression for this case is more complicated than the exact.

Also solved by J. V. BALCH, HORACE OLSON, A. R. NAUER, PAUL CAPRON, and LOUIS G. POOLER.

2666. Proposed by W. WOOLSEY JOHNSON, Annapolis, Md.

Ten equations between five quantities, x_1, x_2, x_3, x_4, x_5 being written as follows: $x_1 = 1 - x_3x_4$ and four others formed by the cyclic interchange of the suffixes; also $x_5x_1x_2 = x_5 + x_2 - 1$ and four others formed by the cyclic interchange; prove that only three of these equations are independent. In other words, the values of x_1 and x_2 being assumed at pleasure, x_3, x_4 , and x_5 can be so determined as to satisfy all ten equations.

I. SOLUTION BY ARNOLD DRESDEN, University of Wisconsin.

Denote by E_i the function $x_i + x_{i+2}x_{i+3} - 1$ and by $\bar{E}_i$ the function $x_ix_{i+1}x_{i+2} - x_i - x_{i+2} + 1$. Then the proposed equations may be written in the form $E_i = 0$, $\bar{E}_i = 0$, ($i = 1, \dots, 5$), the subscripts being reduced modulo 5.

We have the following identities:

$$(1) \quad \begin{cases} x_{i+1}E_i - E_{i+3} = \bar{E}_{i+1} \\ x_{i-1}E_i - E_{i+2} = \bar{E}_{i+2}, \end{cases}$$

so that the equations $\bar{E}_i = 0$ are dependent upon the equations $E_i = 0$.

But from the relation (1), we derive, moreover, the following

$$(5) \quad E_{i-1} + x_{i-1}E_i = x_{i+2}E_{i+1} + E_{i+2}.$$

Hence from any three of the equations, $E_i = 0$, the remaining two must follow.

II. SOLUTION BY HORACE L. OLSON, Heidelberg University, Tiffin, Ohio.

The ten equations mentioned are

$$\begin{array}{ll} (1) & x_1 = 1 - x_3x_4 & (6) & x_5x_1x_2 = x_5 + x_2 - 1 \\ (2) & x_2 = 1 - x_4x_5 & (7) & x_1x_2x_3 = x_1 + x_3 - 1 \\ (3) & x_3 = 1 - x_5x_1 & (8) & x_2x_3x_4 = x_2 + x_4 - 1 \\ (4) & x_4 = 1 - x_1x_2 & (9) & x_3x_4x_5 = x_3 + x_5 - 1 \\ (5) & x_5 = 1 - x_2x_3 & (10) & x_4x_5x_1 = x_4 + x_1 - 1. \end{array}$$

Let us assume, first, that $x_1x_2 \neq 1$. Then from equations (4), (2), and (3), respectively, we find that,

$$x_4 = 1 - x_1x_2, \quad x_5 = (1 - x_2)/(1 - x_1x_2), \quad x_3 = (1 - x_1)/(1 - x_1x_2).$$

To show that the remaining equations are dependent upon these three, we note that equations (3), (4), (2) lead respectively to

$$1 - x_2x_3 = 1 - x_2 + x_1x_2x_5, \quad 1 - x_2x_3 = 1 - x_2 + x_5(1 - x_4), \quad 1 - x_2x_3 = x_5.$$

This last equation is equation (5).

Similarly we have

$$1 - x_3x_4 = 1 - x_4 + x_4x_5x_1, \quad 1 - x_3x_4 = 1 - x_4 + x_1 - x_1x_2, \quad 1 - x_3x_4 = x_1,$$

This last equation is (1).

Multiplying (4) by x_5 , we have $x_5x_1x_2 = x_5 - x_5x_4$ and then $x_5x_1x_2 = x_5 + x_2 - 1$ from (2).

This last equation is (6). Since, then, equations (2) to (5) and (7) to (10) are obtained from (1) and (6) by cyclic permutation of the suffixes, it follows that upon equations (2), (3), and (4) all the others depend. This, we note, is true whether or not $x_1x_2 = 1$. If $x_1x_2 = 1$, we find from (4), (1), and (2), respectively, $x_4 = 0$, $x_1 = 1$, $x_2 = 1$. Equations (3) and (5) then both become $x_3 + x_5 = 1$. Under these conditions equations (6) to (10) are satisfied identically, and x_3 and x_5 are not determined.

Also solved by ELIJAH SWIFT, PAULINE SPERRY, C. C. YEN, and HERBERT N. CARLETON.

QUESTIONS AND DISCUSSIONS.

SEND ALL COMMUNICATIONS TO U. G. MITCHELL, University of Kansas.

DISCUSSIONS.

I. CONCERNING DIRECTION COSINES AND HESSE'S NORMAL FORM.

By MAXIME BÔCHER, Harvard University.

For many years I have felt that Hesse's form for the equation of a line is out of place in a course on elementary analytic geometry¹ and have omitted it from my own course. I am glad to find the same view expressed by two writers in recent numbers of the MONTHLY (p. 476, 1917 and p. 181, 1918) who give proofs of the formula for the distance from a point to a line which do not involve the use of Hesse's form. Perhaps I may be permitted to call attention to a proof of this kind which I have published elsewhere² and which seems to me to have two advantages over those recently given in the MONTHLY, namely, *first*, that it requires no reference to the figure with the consequent necessity (if we wish to be complete, that is, really to give a *proof*) of examining exhaustively the various forms the figure may take on; and *secondly*, that it admits of immediate extension to the corresponding problem in solid analytic geometry.

It is perhaps not quite so clear that Hesse's form for the equation of a plane in solid analytic geometry should be removed from the pages of elementary text books or relegated to small-type sections, since this form is commonly used not merely in connection with the formula for the distance from a point to a plane but also as the primary form of the equation of a plane from which the other forms are deduced. I will indicate how I am accustomed to dispense with it altogether, and at the same time to relegate the too pervasive direction cosines

¹ It has its definite, though very limited, place in higher work.

² Bôcher, *Plane Analytic Geometry*, Henry Holt & Co., 1915, p. 38.